

ANSWERS TO THE  
ODD-NUMBERED PROBLEMS

**Chapter 1, page 5**

1. 10.

3. 1.

$$5. \begin{array}{cccc} 1 & 2 & 1 & \\ & 1 & 2 & 1 \\ 1 & 3 & 3 & 1 \end{array}$$

7.  $1, 1 + 5 = 6, 5 + 10 = 15, 10 + 10 = 20, 10 + 5 = 15, 5 + 1 = 6$ , and 1;  
 $1, 1 + 6 = 7, 6 + 15 = 21, 15 + 20 = 35, 20 + 15 = 35, 15 + 6 = 21, 6 + 1 = 7$ , and 1.

$$9. \binom{9}{7} \cdot \binom{9}{2} = 36, \binom{9}{8} \cdot \binom{9}{1} = 9.$$

$$11. 125x^3 + 150x^2y + 60xy^2 + 8y^3.$$

$$17. c = 56, m = 5.$$

19. (a) 4. (b) 8. (c) 16. (d) 32. (e) 64. (f) 128.

21. (a) 0. (b) 0.

23. (a) 8. (b) 16. (c) 32. (d) 64.

$$25. r = 6, s = 7, t = 6, u = 7.$$

$$27. \binom{996}{990}.$$

$$31. x^4 \% 4x^3y \% 6x^2y^2 \% 4xy^3 \% y^4 \% 4x^3z \% 12x^2yz \% 12xy^2z \% 4y^3z \% 6x^2z^2 \% 12xyz^2 \% 6y^2z^2 \% 4xz^3 \% 4yz^3 \% z^4.$$

33. 3.

37. 3.

39. 36, of which 27 are odd.

41. 0, 1, 3, 7, and 15.

**Chapter 2, page 14**

3. 103.

7.  $F_{n+4} = 3F_{n+1} + 2F_n$ .

9. 321.

13. 702.

19. (a)  $F_{16} - 1$ . (b)  $F_{19} - 1$ .

21. (a) 2. (b) 3. (c) 5. (d) 8. (e) 13. (f) 21.

23.  $r = 4$ ,  $s = 5$ , and  $t = 11$ .

25.  $r = 15$ ,  $s = 17$ , and  $t = 19$ .

29. 204

33.  $L_{2m+2} - 2$ .

35.  $r = 4$ ,  $s = 6$ , and  $t = 8$ .

37. (a) 1. (b) -1. (c) 1. (d) -1.

**Chapter 3, page 18**

1. (a) 5040. (b) 36. (c) 362,880. (d) 720.

5. (a)  $5!$ . (b)  $7!$ . (c)  $(n + 1)!$ .

7.  $a = 14$  and  $b = 10$  or  $a = 24024$  and  $b = 24023$ .

9.  $n^4 + 10n^3 + 35n^2 + 50n + 24$ .

11. (a) 23. (b) 119. (c) 719.

15.  $a = 6$ ,  $b = 11$ , and  $c = 5$ .

17.  $(2m + 2)! - 1$ .

19. (a)  $(n + 2)! + (n + 1)! - 2$ .

**Chapter 4, page 23**

1. (a) -4, 3, 10. (b) -19, -23, -27.

3. (a) 990. (b) 993. (c) 988.

5.  $33/2$ .

7. (a) -33,698. (b) 501,994. (c)  $-11,385/4$ . (d)  $n^2$ .

9. 54, 1458, 13122.

11. 686, 4802, 33614.

13.  $\pm 42$ .

15. (a)  $(7^{1000} - 1)/6$ . (b)  $(1 - 7^{1000})/8$ . (c)  $(7^n - 1)/6$ .

17. 9.

21. (a)  $6\sqrt{3}$ . (b)  $6\sqrt{3}$ . (c) 4. (d) 4.

23. 30.

27.  $(x^{n+1} - y^{n+1})/(x - y)$ .

29. (a) 5. (b) 24 miles per hour.

31.  $2[1 - (1/2)^7]$ .

35. (a) 0, 3, 5, 6, 9, 10, 12, 15, 18, 20, 21, 24, 25, 27, 30.

(b) 15. (c) 15. (d)  $16 \bullet 15 = 240$ .

**Chapter 5, page 35**

13. (a)  $1 \& \frac{1}{n \% 1}$ . (b)  $\frac{x^{2n\%1} \& y^{2n\%1}}{x \& y}$ .

15.  $(n + 1)/2n$  for  $n > 1$ .

21. Using mathematical induction, one can show that:

(a)  $F_{n+1}^2 + F_n F_{n+2} = (F_n)^2$ .

(b)  $\frac{F_{n+2}^2 + F_{n+1}^2}{F_n} = F_{n+3}$ .

(c)  $F_{n-1} + F_{n+1} = L_n$ .

**Chapter 6, page 47**

1. (a) 4,845.      (b) 3,003.      (c) -2,912.

3.  $a = -3, b = 2, c = 0$ .

7. 1, -1, 1, -1, 1, -1.

11.  $\binom{m}{r} = (-1)^r \binom{r+m-1}{m-1}$ .

15.  $(d/2)n^2 + [a - (d/2)]n$ .

19.  $\prod_{i=1}^2 \binom{2}{i} a_i \prod_{i=1}^2 \binom{2}{i} b_i = a_1 b_1 + a_1 b_2 + a_2 b_1 + a_2 b_2$  and  $\prod_{i=1}^2 (a_i b_i) = a_1 b_1 + a_2 b_2$ . These are not always equal, since, for example, they are unequal for  $a_1 = a_2 = b_1 = b_2 = 1$ .

23.  $(1/6)n^3 - (1/2)n^2 + (1/3)n$ .

25.  $n^3 + 5n$ .

27.  $s = 3, t = 1$ .

29.  $r = 6, s = 7, t = 1$ .

31.  $(1/5)n^5 + (1/2)n^4 + (1/3)n^3 - (1/30)n$ .

37.  $(n^3 + 3n^2 + 2n)/6$ .

### Chapter 7, page 55

1.  $a, b; a, c; a, d; a, e; b, c; b, d; b, e; c, d; c, e; d, e$ .

3.  $a, b; b, a; a, c; c, a; a, d; d, a; a, e; e, a; b, c; c, b; b, d; d, b; b, e; e, b; c, d; d, c; c, e; e, c; d, e; e, d$ .

5. 7,920.

7. 30.

9.  $\binom{12}{3}\binom{9}{7}, \binom{12}{3}\binom{9}{2}, \binom{12}{7}\binom{5}{3}, \binom{12}{7}\binom{5}{2}, \binom{12}{2}\binom{10}{7}, \binom{12}{2}\binom{10}{3}$ .

11. -1,201,200.

17.  $3^{100}$ .

19.  $1,2,3,4; 2,1,4,3; 3,1,2,4; 4,1,3,2;$   
 $1,3,4,2; 2,4,3,1; 3,2,4,1; 4,3,2,1;$   
 $1,4,2,3; 2,3,1,4; 3,4,1,2; 4,2,1,3.$

23. (a) 200. (b) 10,150.

### Chapter 8

#### Sections 8.1, 8.2 and 8.3, page 64

1.  $(x + 2)(x^3 + 3x^2 - 6x + 2) - 16$ .

3.  $\pm 1 \pm \sqrt{3}$ .

5.  $a = -27,604$ .

7.  $\pm 1, 3, \sqrt{2}i, \&\sqrt{2}i$ .

9.  $f(0) = 0 = f(a)$ . Two factors are  $x$  and  $x - a$ .
11. 0, -2, -4, -2/3.
13.  $\frac{5}{3}, \frac{\sqrt{17}}{2}, \frac{\sqrt{17}}{2}$ .
15. 0,  $\frac{1}{3}, \frac{2}{3}, \frac{1 + \sqrt{3}i}{6}, \frac{1 + \sqrt{3}i}{6}$ .
17.  $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$ .
25.  $x^8 - 48x^6 + 536x^4 - 1728x^2 + 400 = 0$ .
27. (a)  $b$ .  
 (b)  $c(2x + 1) + b$ .  
 (c)  $d(3x^2 + 3x + 1) + c(2x + 1) + b$ .  
 (d)  $\binom{n}{1}x^{n-1} + \binom{n}{2}x^{n-2} + \binom{n}{3}x^{n-3} + \dots + \binom{n}{n-1}x + \binom{n}{n}$ .
29. (a) 0. (b) 0. (c)  $6d$ .
33.  $a = 38, b = 84, c = 81, d = 34, e = 5$ .
39. No.

**Section 8.4, page 68**

1. (a) 4. (b) 8/3. (c) 16. (d) 32/3. (e) 16/3. (f)  $\frac{4\sqrt{3}}{3}$ .
5. 1.
7.  $a = p^3 - 2pq, b = p^4q - 3p^2q^2 + 2q^3, c = p^3q^3 - 2pq^4, d = q^6$ .
9. (a) 0. (b) -3. (c) 1. (d)  $y^3 - 3y - 1 = 0$ .
11. (a)  $x = 3, y = -3, z = 1$ .  
 (b)  $t = 4, u = 2, v = -4, w = 1$ .

## Chapter 9

### Sections 9.1 and 9.2, page 78

1.  $x = -6/53, y = -91/53.$
3.  $x = 103/21, y = -71/21, z = -4/63.$
5.  $x = 5, y = 2, z = 4.$
7.  $x = (b + c - a)/2, y = (c + a - b)/2, z = (a + b - c)/2.$

### Sections 9.3, page 85

15.  $4xyz.$
17. (b)  $x = -1, y = 2, z = 0.$   
(c)  $x = 2, y = -3, z = 3, w = 0.$

(d)

$$L_{2n} = \frac{3L_n^2 + 2L_n L_{n+1} + 2L_{n+1}^2}{5},$$

$$L_{3n} = \frac{2L_n^3 + 3L_n^2 L_{n+1} + 3L_n L_{n+1}^2}{5}.$$

### Section 9.4, page 90

5.  $(w - x)(w - y)(w - z)(z - x)(z - y)(y - x).$
7.  $(w - x)(w - y)(w - z)(z - x)(z - y)(y - x).$
9.  $r = 2, s = 2, t = 1.$
11.  $r = 4, s = 2, t = 2.$
13. (b)  $x = 1, y = 1, z = 1.$

## Chapter 10

### Section 10.1, page 95

19.  $a = 2, b = 3, c = 4.$

### Sections 10.3 and 10.4, page 103

21. (a)  $101/100.$

23. (a) The arithmetic mean is 2; the geometric mean is  $\sqrt[4]{2}.$

(b)  $a = 15, b = -20, c = 15, d = -6.$

(c) The roots are equal, and so all equal to 1, since their arithmetic mean is the same as their geometric mean.

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